



# Optimal load distribution model of microgrid in the smart grid environment



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## ABSTRACT

In the smart grid environment, the flexible and diverse distributed generation (DG) and microgrid (MG) are attracting considerable attention. There are many key management and optimization issues involved in smart grid. As an important part of smart grid optimizations, the optimal load distribution of MG contributes to the efficient operation of MG in the smart grid environment. However, traditional optimal load distribution models of large-scale power generation systems are not fully applicable to MG for their distinct characteristics. In this paper, we first introduce the MG in smart grid and analyze its characteristics. Then, we present a review of the optimal load distribution models of MG in the smart grid environment, and point out the deficiencies of the existing models. Finally, a comprehensive optimal load distribution model of MG both in objective functions and constraints is established and discussed.

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## 1. Introduction

With the rapid development of power generation technologies and the increasing demand for electricity, large-scale centralized power generation systems were widely implemented, in which thermal power is the major power generation method. However,

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the disadvantages of large-scale centralized thermal power generation, such as disadvantages in the aspects of economic efficiency, environmental protection, security and reliability, and control agility, were gradually emerging. People began to re-evaluate the large-scale centralized power generation methods, and the idea of small-scale distributed generation was presented [1]. After the late 1990s, increasing research efforts have focused on the distributed generation (DG) and its economic benefits [2–5]. DG, also known as dispersed generation or distributed energy supply, is a new energy production and supply model [6–8]. In DG, relatively small power generation units are arranged near the user load sites. Therefore, compared with the large-scale centralized power generation, the main advantages of DG are its dispersion in locations and flexibility in power supply. In addition, DG can also improve the reliability of power supply system when collaborating with large-scale power generation systems [9]. However, DG also has some disadvantages. The main power grid always employs methods like restriction and isolation to reduce the impact from DG. With the development of smart grid, the concept of microgrid (MG) [10–12] was proposed in order to coordinate the contradictions between DG and the main power grid. MG is an important power generation mode in smart grid [13–15].

Currently, MG is primarily in the experimental research stage [16–18]. There are still many obstacles in terms of technology, control and management that needs to be overcome before MG can be widely applied [19–21]. The economic optimization of MG is one of the key management issues which can affect its operating efficiency and user acceptance [22–24]. Three aspects of MG economics were summarized in [25]. Solar power, wind power and other renewable energy power generation methods are widely adopted in MG, which makes its environmental benefits apparent [26]. Also, power generated in MG can be supplied in a safer, more reliable, and more personalized way by means of electricity users division and load classification [27,28]. Currently, there have been some research efforts in the economic optimization of MG [29–32].

The various optimization issues are the key parts of MG economics, such as the location optimization [33–35] and capacity optimization [36–38] of distributed generators in order to determine the location and capacity of each generator. The optimal load distribution is an important optimization problem to support the efficient operation of MG [39]. The optimal load distribution of MG is to achieve multi-objectives, including low operating cost, low emissions, high reliability, high power quality, and low line loss, etc., while meeting various system operating constraints. In this paper, we will review some existing optimal load distribution models of MG, and point out the deficiencies of the existing models. Then a general and comprehensive model of MG optimal load distribution is proposed and discussed.

The remainder of this paper is structured as follows. A brief introduction of smart grid and MG, as well as the optimal load distribution of MG are described in Section 2. Then, the optimization mathematical models of MG optimal load distribution, both single-objective and multi-objective, are reviewed in Section 3. In Section 4, we propose a comprehensive model of MG optimal load distribution, including different objective functions and various constraints. Finally, conclusions are drawn in Section 5.

## 2. Optimal load distribution of MG in the smart grid environment

### 2.1. MG in the smart grid

To achieve the goal that the operation of power systems becomes safer, more reliable, more environmentally friendly, more

flexible, more controllable, and more cost-effective, the United States and some European countries proposed the concept of “smart grid” [40–42]. Currently, many countries have put smart grid as one of the national strategies, and considerable research work have been carried out [43–45]. Smart grid [46–48] can be generally considered as an intelligent power grid system which integrates the energy flow and information flow by advanced information technology, sensor technology, automatic control technology and scientific management methods, etc.

MG is an important part of smart grid implementation and application. A variety of distributed generators and energy storage devices can assess to the main power grid in the smart grid environment. They can form a “virtual power plant” through the interconnection on all voltage levels by electrical communication systems. Many researchers studied the roles of distributed generators and energy storage devices in smart grid. Zhang [49] described a framework for the operation and control of smart grid with DG and Flexible AC Transmissions (FACTS), and proposed a global coordinated strategy for voltage control. Mohd et al. [50] pointed out that distributed energy storage systems with advanced power electronics can play an important role in power supply systems and lead to many financial benefits. They also studied the topologies, control and flexibility of the main power grid with energy storage systems. Chowdhury et al. [51] studied the operation and control of distributed generators in power islands by simulation, and an appropriate bus controller was designed. They also analyzed the flexibility, safety and power quality of DG. These research efforts mainly focused on the technical and control perspective of DG operation in the smart grid environment.

### 2.2. The characteristics of MG optimal load distribution

MG is a kind of small power distribution and consumption system, which overcomes the deficiencies of DG in terms of intelligence and flexibility [52]. MG provides power energy for users by modern power techniques, such as fast power electronic switches, advanced converting techniques, efficient new energy sources and various energy storage devices. Many kinds of distributed generators and energy storage devices, including photovoltaic (PV) arrays, wind turbines (WTs), microturbines (MTs), fuel cells (FCs), and energy storage batteries, are widely adopted in MG.

MG is a controllable power system that provides thermal and electricity for users by combining the power generation equipment, energy storage devices, control equipment and load [53]. The basic structure of MG has been presented by the America Consortium for Electric Reliability Technology Solution (CERTS) [54]. MG can operate in both grid-connected mode by collaborating with main power grid and independent isolated mode. As an effective supplement of main power grid, MG has attracted considerable attention in recent years due to its characteristics of low cost, low pollution, high reliability and easy to control.

The optimal load distribution of MG is to achieve multiple goals, including minimizing cost, minimizing pollution emissions, and maximizing reliability, etc., while satisfying various constraints. Fig. 1 shows the basic structure of MG optimal load distribution.

There are many differences between the MG and traditional large-scale thermal power generation. First, power generation of various distributed generators in MG usually follows the maximum power-point tracking mode [55,56], and cannot be controlled through manual scheduling. The output characteristics of these distributed generators of renewable energy power generation are sensitive to the natural conditions, such as the intensity of sunlight and the velocity of wind. Second, the power

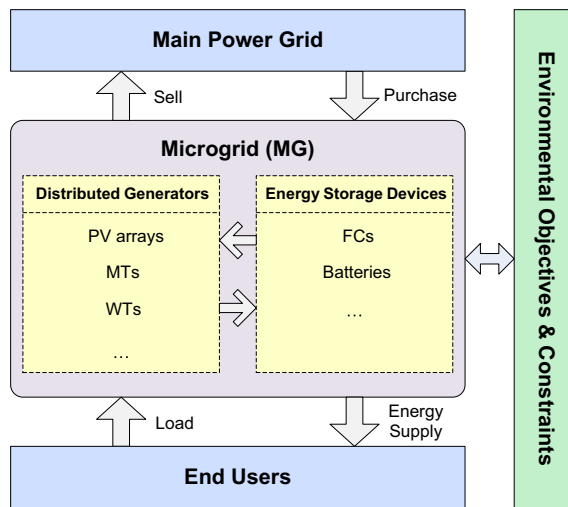


Fig. 1. Basic structure of MG optimal load distribution.

line loss of MG cannot be ignored since the wiring resistance caused by the low voltage is large. Third, the equal incremental principle [57] cannot be directly used for MG, since the power generation characteristics of distributed generators in MG are different from traditional thermal power units. Also, MG should be optimized globally and dynamically according to the system operation, since the power distribution of MG may change a lot when converting between grid-connected mode and isolated mode [58].

Therefore, traditional optimal load distribution models of the large-scale power systems [59–61] cannot be directly applied to MG due to its specific characteristics. New optimal load distribution models for MG need to be developed.

### 3. Optimal load distribution models of MG

#### 3.1. Single-objective models

The optimal load distribution of MG is actually a multi-objective optimization problem. However, in some existing research work, the multi-objective optimal load distribution models of MG were always simplified to single-objective ones in order to facilitate problem analysis and solving. Chen and Zhu [58] established a single-objective model of MG optimal load distribution. The objective of their model is to minimize the total cost of power generation by optimizing the outputs of distributed generators while meeting the system operation conditions. The objective function of their model is defined as follows:

$$\min F = \min \sum_{t=1}^T \left[ \sum_{i=1}^N F_i(P_i(t)) + E_{buy} P_{buy}(t) - E_{sell} P_{sell}(t) \right] \quad (1)$$

where  $F$  is system total cost of power generation,  $T$  is the total number of periods in scheduling cycle,  $t$  is the number of period,  $N$  is the total number of power generation units and energy storage devices which can be scheduled within the system,  $P_i$  represents the actual power output of unit  $i$ ,  $F(P_i)$  denotes the operating cost of the  $i$ th power generation unit or energy storage device,  $P_{buy}$  is the electricity bought from the main power grid,  $P_{sell}$  is the electricity sold to the main power grid,  $E_{buy}$  is the price of bought electricity, and  $E_{sell}$  is the price of sold electricity.

There are four constraints in this optimization model, including the operation constraints of power generation units, charge–discharge and capacity constraints of energy storage devices, and power balance constraints. The total operating cost in this model

includes fuel cost, maintenance cost, start-up cost, and trading cost and income with main power grid, but does not include the factor of fuel consumption rate. Moreover, the objectives of pollution emissions, system reliability, power quality, and operation efficiency were not considered in the model. Also, the time and quantity constraints of start-stop were not included as well. Therefore, the simulation results of this model may not be so accurate.

Hernandez-Aramburo et al. [62] pointed out that the most significant difference between MG and large-scale power generation systems is the presence of a local heat demand, which adds another dimension to the optimization problem of MG. They proposed an optimal load distribution model that considered the Combined Heat and Power (CHP) System. The aim of their model was to reduce the fuel consumption rate while meeting the local energy demand (both electrical and thermal). However, the emissions and the maintenance costs were not included in the objective function of their model. Therefore, it is difficult to balance the relationship between cost and emissions when using their model.

Asanol and Bando. [63] and Asano et al. [64] also studied the optimal load distribution of MG considering CHP system. The objective function of the mathematical model proposed in [63] includes operating cost, gas cost, electricity bought cost and start-up cost of gas engine. The objective function proposed in [64] takes more costs into consideration, including gas costs, transaction costs with main power grid, the contract demand cost from the main power grid, operation and maintenance costs, start-up costs and the original investment costs. The constraints of this model include power supply and demand balance, charge–discharge constraints of energy storage devices, space cooling constraints, heat gas and hot water demand constraints. However, both the two models [63,64] were simplified to single-objective optimization problems. Moreover, capacity constraints of energy storage devices and operation constraints of power generation units were not included.

Zoka et al. [65] studied the optimal operational issues of distributed generators and energy storage devices from the user perspective, and proposed an economic evaluation method for independent network of distributed generator resources. They also pointed out that the total cost of MG should include not only the installation and operating costs, but also self-construction cost and power interruption cost which are the measures of MG reliability. The total cost in the objective function is composed of operating costs, investment cost, and the revenue/costs of the transactions with main power grid. The operating costs consist of fuel costs and inspection and maintenance costs. The constraints in the model include power supply and demand balance constraint, heat supply and demand balance constraint, charge–discharge balance constraint of energy storage devices, generation capacity constraint, the constraints of other distributed generators, and non-negative constraints. The advantages of this model are twofold. First, its objective function integrates the concept of power interruption cost, which indicates the reliability of MG system. Second, the constitution of both its objective function and constraints is more comprehensive. However, the start-up costs and the line loss are ignored in the objective function. Also, energy storage capacity constraint is not included in the constraints.

In summary, all of the abovementioned MG optimal load distribution models were simplified to single-objective optimization problems. However, the optimal load distribution of MG is a multi-objective optimization problem in which we should not only consider the objective of costs, but also take the objectives of emissions, reliability, power line loss, power quality and power generation efficiency into account. These objectives are of great importance to the precise and effective optimal load distribution

of MG. Therefore, subjectively ignoring some of these objectives to simplify the multi-objective model into single-objective ones is unreasonable, which can directly affect the accuracy and effectiveness of the MG optimal load distribution results.

### 3.2. Multi-objective models

Mohamed and Koivo [66–69] studied the optimal modeling and online management of MG, and established a series of multi-objective nonlinear mathematical models of MG optimal load distribution, in which the minimizing of costs, emissions and the trade-off between them were considered at the same time. The operating costs of the multi-objective optimization model established in [66] are composed of fuel costs and operation and maintenance costs, while the start-up cost, revenue and costs with main power grid are not included.

The total emissions of pollutants in [66] are expressed as follows:

$$E(P_i) = \sum_{i=1}^N 10^{-2} (\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \zeta_i \exp(\lambda_i P_i) \quad (2)$$

where  $E(P_i)$  is the emissions of unit  $i$ ,  $\alpha_i, \beta_i, \gamma_i, \zeta_i$  and  $\lambda_i$  are nonnegative coefficients describing the emission characteristics of the  $i$ th unit, and  $P_i$  is the real output of the  $i$ th unit.

The constraints in this model are inadequate, which only include the system power balance constraint and distributed generator operation constraint. Constraints of energy storage devices and system start-up time and quantity are not included. The incomplete model will lead to inaccurate results.

In [67], the authors analyzed the cost characteristics of wind power generation systems, photovoltaic systems, diesel generators, fuel cells, micro turbines, and batteries, and proposed the corresponding cost calculation methods. Then a multi-objective optimization model of MG was established. The emission objective function is the same as Eq. (2), while system start-up cost is added to the operating cost. However, some important factors, such as the revenue and costs with main power grid, are still not considered. For the constraints, the continuous running/stop time constraint and start-stop quantity constraint are added. But the charge-discharge constraint and capacity constraints are not taken into account.

The optimal load distribution model established in [68] is much more complete than the previous two models in [66,67]. The emission objective function is the same as Eq. (2), and operating costs include fuel cost, operation and maintenance cost, start-up cost, and the revenue and costs with main power grid. The operating costs function is presented as follows:

$$F(P_i) = \sum_{i=1}^N (C_i F_i + OM_i + SC_i + CPP_i - ISP_i) \quad (3)$$

where  $F(P_i)$  is the total operating cost of unit  $i$ ,  $P_i$  represents the actual power output of unit  $i$ ,  $N$  is the total number of units in the MG system,  $C_i$  is the fuel costs of unit  $i$ ,  $F_i$  is the fuel consumption rate of unit  $i$ ,  $OM_i$  is the operation and maintenance cost of unit  $i$ ,  $SC_i$  is the start-up cost of unit  $i$ ,  $CPP_i$  is the cost of electricity purchased from the main power grid, and  $ISP_i$  is the income of electricity sales to the main power grid.

Constraints of the model in [68] are the same as those in [67], while the trading between MG and main power grid is considered more comprehensively. Two different conditions are considered to model the purchased and sold electricity.

$$CPP = C_p \times \max(P_L - P_i, 0) \quad (4)$$

$$ISP = C_s \times \max(P_i - P_L, 0) \quad (5)$$

where  $C_p$  and  $C_s$  are tariffs of the purchased and sold electricity

respectively,  $P_i$  is the generated electricity, and  $P_L$  is the load demand.

In [69], the authors added the emission objective measured by emission factor to the total objective function. Operating cost and constraints are the same as those in [68]. This model also has some inadequacies. For example, the reliability, power line loss, power quality, and power efficiency are not considered in the objective function, and the charge-discharge constraint of energy storage devices is not included in the constraints of the model.

Wang et al. [70] studied the optimal management of distributed generation, and presented a multi-objective niche evolutionary immune algorithm (MO-NEIA). The objective function proposed includes electricity quality indicated by voltage deviation, operation efficiency of generators indicated by electricity factor, and power line loss. The constraints include variable constraints (active power, reactive power and voltage constraints), and power flow equation constraints. The main advantage of this model is that it takes the power quality, the operation efficiency of generators, and the power line loss into account, and the quantitative representations of these factors are given. These are rarely considered in other models. However, some costs are still not considered in their objective function, and system power balance constraint, charge-discharge and capacity constraint are not considered in constraint conditions.

Though multi-objective optimization models of MG optimal load distribution have been established in the aforementioned studies, the main objectives are still operating cost and emissions. Many important objectives, such as reliability, power line loss, power quality, power generation efficiency, etc., are still ignored. Meanwhile, the constraints are also inadequate.

Based on the review and analysis of existing studies on MG optimal load distribution, we find that many single- and multi-objective optimization models have been proposed. Some studies simplified the multi-objective optimization problem to single-objective one. Also some multi-objective models established were still inadequate in terms of objective functions and constraints. All of these problems will affect the precision and effectiveness of the optimal load distribution results of MG. Therefore, a more comprehensive model for MG optimal load distribution model is needed.

## 4. A comprehensive model of MG optimal load distribution

### 4.1. Objective function of MG optimal load distribution

Actually, the optimal load distribution of MG is a multi-objective optimization problem. Though the emission of MG is not so much as the traditional thermal power generation since renewable power generations are widely used in MG, we still cannot ignore the factor of emission in the objective function. In addition, a key characteristic of MG is its reliability which can be measured by the power interruption costs. Also, the electricity generated by MG is low voltage, so the power line loss cannot be ignored. Moreover, since transactions exist between MG and the main power grid when power generated by MG is not equal to the load demand, the revenue and cost of transactions between MG and the main power grid should be included in the total operating cost. Finally, power quality and operation efficiency of MG should also be considered. Therefore, the objective function of MG optimal load distribution is defined as follows:

$$\min F(P_i) = \min \{F_1(P_i), F_2(P_i), F_3(P_i), F_4(P_i), F_5(P_i), F_6(P_i)\} \quad (6)$$

$$F_1(P_i) = \sum_{i=1}^N (C_i F_i + M_i + S_i + CE_i + CP_i - CS_i) \quad (7)$$



$$F_2(P_i) = \sum_{i=1}^N E_i \quad (8)$$

$$F_3(P_i) = \sum_{i=1}^N R_i \quad (9)$$

$$F_4(P_i) = \sum_{i=1}^N L_i \quad (10)$$

$$F_5(P_i) = \sum_{i=1}^N Q_i \quad (11)$$

$$F_6(P_i) = \sum_{i=1}^N Eff_i \quad (12)$$

where  $F(P_i)$  is the overall objective of MG system,  $F_1(P_i)$  is the total operating costs,  $F_2(P_i)$  is the total emissions,  $F_3(P_i)$  is the interruption cost which indicates the reliability of MG,  $F_4(P_i)$  is the total power line loss,  $F_5(P_i)$  is the cost of maintaining an acceptable power quality, and  $F_6(P_i)$  is the total power generation efficiency. In each of the objective function,  $P_i$  represents the actual power output from unit  $i$  in the MG system,  $N$  is the total number of units in MG,  $C_i$  is the fuel cost of unit  $i$ ,  $F_i$  is the fuel consumption rate of unit  $i$ ,  $M_i$  is the maintenance cost of unit  $i$ ,  $S_i$  is the start-up cost of unit  $i$ ,  $CE_i$  is the emission cost of unit  $i$ ,  $CP_i$  is the cost of electricity purchased from the main power grid when the load demand exceeds the generated electricity from MG,  $CS_i$  is the income of electricity sold to the main power grid when the generated electricity from MG exceeds the load demand,  $E_i$  is the emission of unit  $i$ ,  $R_i$  is the power interruption cost of unit  $i$  which indicates its reliability,  $L_i$  is the line loss of unit  $i$ ,  $Q_i$  is the cost of maintaining an acceptable power quality of unit  $i$ , and  $Eff_i$  is the operation efficiency of unit  $i$ . For the distributed generators in MG, their operation efficiency “ $Eff$ ” is the efficiency of converting the input energy into electrical energy. For the energy storage devices in MG, their “ $Eff$ ” means the power output efficiency of the stored electrical energy. In general, the operation efficiency of MG refers to the total energy input-output efficiency.

See related references for the calculation of each optimization objective.

## 4.2. Constraints of MG optimal load distribution

### 4.2.1. Power supply and demand balance constraints

**Power system balance constraints.** The MG system should keep balance between power supply and demand. This equality constraint is denoted as follows:

$$\sum_{i=1}^N P_i + \sum_{i=1}^{N_s} ES_i + P_{purchase} - P_{sell} = P_L + L \quad (13)$$

where  $P_i$  is the actual output of unit  $i$ ,  $ES_i$  is the stored energy of energy storage unit  $i$ ,  $N$  and  $N_s$  are the total number of generators and energy storage devices respectively in the MG system,  $P_{purchase}$  is electricity purchased from the main power grid,  $P_{sell}$  is the electricity sold to the main power grid,  $P_L$  is the total load, and  $L$  is the total power line loss.

**Generation capacity constraints.** The actual output of each unit should satisfy its lower and upper output limits as follows:

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (14)$$

where  $P_i^{\min}$  and  $P_i^{\max}$  are the lower and upper power out limits of unit  $i$ , respectively.

### 4.2.2. Operation constraints of energy storage devices

**Charge-discharge constraints.** Energy storage devices play important roles in MG, their operation must satisfy the charge-discharge

constraints.

$$EC_i^{\min} \leq EC_i \leq EC_i^{\max} \quad (15)$$

$$ED_i^{\min} \leq ED_i \leq ED_i^{\max} \quad (16)$$

where  $EC_i$  and  $ED_i$  are charged and discharged amount of energy storage device  $i$ , respectively;  $EC_i^{\min}$ ,  $EC_i^{\max}$  and  $ED_i^{\min}$ ,  $ED_i^{\max}$  are the lower and upper limits of charge and discharge, respectively.

**Capacity constraint.** The capacity of energy storage devices must be in a certain interval range.

$$ES_i^{\min} \leq ES_i \leq ES_i^{\max} \quad (17)$$

where  $ES_i$  is the capacity of energy storage unit  $i$ ,  $ES_i^{\min}$  and  $ES_i^{\max}$  are the corresponding lower and upper limits.

### 4.2.3. Power generation constraints of renewable energy

**PV power constraint.** The output of PV power is significantly affected by natural conditions, such as the temperature and the solar radiation at a certain time and in specific region. The PV power generated cannot exceed the power capacity under strongest nature condition.

$$P_{PV,i} \leq P_{PV}^{\max} \quad (18)$$

where  $P_{PV,i}$  is the power generated by PV unit  $i$ , and  $P_{PV}^{\max}$  is its upper limit.

**Wind power constraint.** Wind velocity has a great impact on wind power generation. The wind power generation is limited as follows:

$$P_{WT,i} \leq P_{WT}^{\max} \quad (19)$$

where  $P_{WT,i}$  is the power generated by wind power unit  $i$ , and  $P_{WT}^{\max}$  is its upper limit.

### 4.2.4. CHP system constraints

**CHP power constraint.** The power generated from CHP system cannot exceed the maximum value, which is expressed as follows:

$$P_{CHP,i} \leq P_{CHP,i}^{\max} \quad (20)$$

where  $P_{CHP,i}$  is the power generated from CHP unit  $i$ , and  $P_{CHP,i}^{\max}$  is its upper limit.

**Continuous running/stop time constraint.** Once the units are switched on, they have to operate continuously for a certain minimum time before being switched off again. On the other hand, they have to maintain the termination status for a certain time before being started again. The violation of these constraints may shorten the life of the equipment [68]. The continuous running/stop time constraints are described as follows:

$$(T_{t-1,i}^{\text{on}} - MUT_i)(\theta_{t-1,i} - \theta_{t,i}) \geq 0 \quad (21)$$

$$(T_{t-1,i}^{\text{off}} - MDT_i)(\theta_{t-1,i} - \theta_{t,i}) \geq 0 \quad (22)$$

where  $T_{t-1,i}^{\text{on}}/T_{t-1,i}^{\text{off}}$  is the on/off time of unit  $i$ ,  $MUT_i$  and  $MDT_i$  are the minimum running and top time limits of unit  $i$  respectively, and  $\theta_{t-1,i}$  represents the on/off status of unit  $i$  represented by 1/0.

**Quantity constraint of start-stop units.** The quantity of start and stop units cannot exceed a certain number.

$$N_{\text{start-stop}} \leq N_{\max} \quad (23)$$

where  $N_{\text{start-stop}}$  is the number of start-stop units, and  $N_{\max}$  is the upper limit.

### 4.2.5. Other constraints

**Reliability constraint.** The failure of MG units will lead to the system outage. Taking the outage probability of MG into account,

the reliability constraint is defined as follows:

$$P_{ml} \leq \alpha P_L \quad (24)$$

where  $P_{ml}$  is the load that may loss,  $P_L$  is the total load, and  $\alpha$  is the probability coefficient.

**Emissions constraint.** The distributed generators or energy storage devices that use fossil fuel will generate a certain amount of emissions. Emissions of MG must be limited in a certain interval range according to related environmental protection policies. Emissions constraint is expressed as follows:

$$\sum_{i=1}^N Y_{ij} \leq Y_j^{\max} \quad (25)$$

where  $Y_{ij}$  is the amount of pollutant  $j$  generated by unit  $i$ , and  $Y_j^{\max}$  is the upper limit of pollutant  $j$ .

**Voltage deviation constraint.** The voltage deviation, which is typically a measure of power quality, of distributed generators and energy storage devices in MG cannot exceed an upper bound, which is described as follows:

$$\Delta V_i \leq \Delta V_{\max} \quad (26)$$

where  $\Delta V_i$  is the voltage deviation of unit  $i$ , and  $\Delta V_{\max}$  is the upper limit.

**User defined constraints.** In addition to the aforementioned various constraints. We should also note that emissions and efficiency in different equations may result in dependent rows in the array to solve. Therefore, in practical applications, this kind of user defined constraints should also be considered.

## 5. Conclusion

The optimal load distribution of MG is an important aspect to achieve its optimal operation and efficient application in the smart grid environment, and the establishment of optimization mathematical models is the first step of MG optimal load distribution. We first review some existing optimal load distribution models of MG, and pointed out the shortcomings of these models. Then we propose a general and comprehensive model. It is of great importance to improve the accuracy and effectiveness of MG optimal load distribution models, thereby promoting the study of MG economics and achieving the intelligent power generation and distribution of smart grid.

We should note that the selection of an appropriate MG optimal load distribution model in practical smart grid applications is important. The model established in this paper is a general model, and we should consider the specific characteristics while using the model. In addition, it is important to explore more effective and efficient algorithms to solve the optimal load distribution problems of MG after the model was established. Some of the traditional optimization methods [71–73] may not be suitable for MG optimal load distribution, which is a nonlinear, multi-objective and multi-constraint optimization problem. Some intelligent algorithms, such as genetic algorithms, bee colony optimization, and particle swarm optimization algorithm, are promising methods to solve the MG optimal load distribution problems [74–78].

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